

# A model of parallel time estimation

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## Abstract

In earlier work, Taatgen, Van Rijn and Anderson (in press) have shown that embedding a simple module that generates temporal information in a more general cognitive architecture explains timing phenomena that were earlier attributed to a hypothesized more complex temporal system. However, the embedded temporal module does not support parallel time estimations, of, for example, two concurrent estimations. Explaining the human capacity of doing multiple time estimations requires either adding additional temporal modules, or assuming higher level processing to strategically use a single timer for parallel timing. This paper presents an experiment and a computational model that show that the latter approach is more plausible for human parallel time estimation.

**Keywords:** parallel time estimation; temporal processing; temporal arithmetic; ACT-R; cognitive modeling.

## Introduction

Timing is an essential aspect of human behavior, both inside and outside psychological laboratories. How long does one wait before pressing the back-button in an Internet browser when the requested page does not appear? How does one know how time passes during a lexical-decision experiment in which one has to be as fast and accurate as possible? Several theories have tried to describe the nature of time estimation both in terms of brain areas and processes (e.g., Buhusi & Meck, 2005), and in terms of behavior (e.g., Gibbon, 1977). Although accounts differ on the exact mechanisms, all assume time is perceived on a logarithmic scale, which means that two longer intervals that differ in duration by a certain amount of time are considered to be more similar than two shorter intervals differing by the same amount of time. In other words, perceived differences grow smaller as the duration increases. Because a logarithmic scale needs a starting point, a start signal is needed to indicate the beginning of an interval. The logarithmic scale is reflected in empirical observations such as the scalar property (the variance of a time-estimation distribution is linearly related to the duration of the estimated time, Gibbon, 1977) and bisection phenomena (a duration exactly in between a long and a short interval is more often considered long than short, Allan & Gibbon, 1991).

The requirement of a start-signal and a logarithmic scale raises the question whether and how multiple overlapping time intervals can be measured. Several studies have

investigated this question, and have concluded that both animals and humans are capable of estimating multiple intervals (Meck & Church, 1984; Ivry & Richardson, 2002; Brown & West, 1990; Penney, Gibbon & Meck, 2000). These studies either seem to suggest or are the basis for claims that there are multiple clocks that can operate in parallel. However, an alternative explanation is that there is only a single clock that is intelligently used by the cognitive system to estimate multiple intervals in parallel. To test this hypothesis, we designed the experiment presented below. In this experiment, participants have to produce two time intervals that overlap partially. They receive a start signal for one of the intervals, and after a SOA of 500-1500ms the start signal for the second interval. They then have to respond to each of the intervals at the appropriate moment. The random interval between the two start signals prevents fixed timing strategies, and produces a variable overlap between the two intervals (Figure 1).

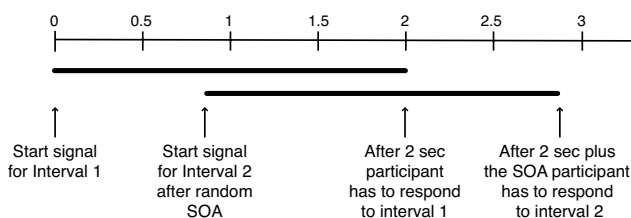


Figure 1: Experimental paradigm used in the experiment

Based on our approach to time perception, which we will discuss in more detail later, two accounts of multiple, parallel time estimation can be proposed. One proposal is to have multiple timers that can be used independently<sup>1</sup>, an account similar to that suggested by Meck and Church (1984). If humans can recruit multiple parallel timers, the timers themselves should not produce any decrease in performance when multiple parallel intervals have to be estimated. A decrease in performance, however, can be due to other factors like attention, dual-tasking costs, the need to learn multiple different intervals, etc. These factors are either independent of the overlap between the two intervals, or, in the case of dual-tasking costs, increase with the

<sup>1</sup> Note that this does not necessarily imply multiple neural clocks. It might be that a single time source, for example an oscillating group of neurons, is driving multiple estimations.

amount of overlap. The general finding is that multi-tasking increases the length of time estimations (Block & Zakay, 1997). The multiple timer account would therefore predict either complete independence of the estimates, or an increase of the estimates with an increased amount of overlap between the intervals.

A second proposal is a single source of time information that can be used strategically by the general cognitive architecture to estimate multiple parallel time intervals sequentially. To produce the two intervals in Figure 1, the SOA between the two start signals has to be remembered during the production of the first interval. After the response on the first interval has been made, one has to wait for a duration equal to the remembered SOA before making the second response. The consequence of this method is that estimates are no longer independent. For example, if the first estimate is too long we also expect the second estimate to be too long. A second consequence of serialization is that the logarithmic time scale will bias the second estimate. The 800ms between the onset of stimulus one and two is internally represented on the logarithmic scale, resulting in, for example, an internal length of 5. When this internal representation is added to the first interval to estimate the second interval, this length of 5 represents a longer time than was earlier perceived. This temporal discounting results in overestimates of the second interval, which becomes larger as the SOA increases. A larger SOA leads to a smaller overlap between intervals, so where the single-timer account predicts longer estimates for larger SOAs (because of the logarithmic scale), the multiple-timer account predicts either no effect (no multitask penalty) or shorter estimates for large SOAs than for short SOAs (assuming a multitask penalty).

## Experiment

### Method

**Subjects** Twenty-six students from Carnegie Mellon University were paid \$8 for participation in the experiment. Five participants were excluded from further analysis because of not adhering to the instructions.

**Design** The experiment consists of two blocks. The purpose of the first block was to have the participants learn a solid and correct representation of the to be estimated time. Hereto, no parallel timing was necessary in this block: participants were required to estimate just a single interval per trial. For each trial, correctness feedback was given. In the second block, this learned duration had to be reproduced twice per trial as illustrated in Figure 1. The main manipulation in this second block was the stimulus onset asynchrony.

**Stimuli & Procedure** In the first block of 46 trials, participants were asked to estimate an unspecified interval. Each trial started with a colored circle being shown on

either the left or the right side of the screen. The participants were instructed to press a key when the presented stimulus was on the screen for the to be estimated time. For the left, green stimulus the “z” had to be pressed, for the right, blue stimulus the “/”. During 6 startup trials, their response was plotted on a timeline where an area was marked as correct. This way, participants could infer whether they had to lengthen or shorten their estimated durations. The marked region ranged from 1750 to 2250ms, making a response of 2000ms optimal. After the six startup trials with timeline feedback, feedback was only given in terms of “correct”, “too fast” or “too slow”. In the second, experimental block of 120 trials, participants had to respond to both left and right stimuli in each trial. Either stimulus appeared first in half of the trials. The stimulus onset asynchrony (SOA) was randomly sampled from the interval  $a = <500, 900>$  or  $b = <1100, 1500>$ . For both stimuli, feedback was given as “correct”, “too fast” or “too slow”.

### Results and Discussion

The first block of trials was presented to internalize the to be estimated duration and to assess single estimation performance. The average estimated duration in the last ten trials of the first block was 1930ms (SD=383). Note that given the positive skew in the distribution of time estimations, an estimated time shorter than 2000ms is optimal. The proportion of correct responses is .583, which is close to the expected proportion given the used correctness-range.

The proportion of correct responses in the second, experimental block was .448 for the first estimation and .435 for the second estimation. This difference is not significant ( $t(20)=0.59$ ). When compared with Block 1, both first and second estimations of Block 2 show worse performance than during the last phase of Block 1 ( $t(20)=2.46$ ,  $p = 0.023$  and  $t(20)=2.72$ ,  $p=0.013$  respectively). At first sight, this seems to be in line with a multiple independent clocks account with a dual-tasking penalty. However, another prediction of this account is that the amount of overlap – and therefore the SOA – should influence the accuracy. For short SOAs, associated with a longer overlap and therefore a higher dual-tasking penalty, the accuracy should be lower than for long SOAs, which result in a shorter overlap. To test this, we compared two linear mixed effect (LME) models (Bates, 2005, and see for a non-technical introduction, Baayen et al, submitted). The first model, predicting accuracy of the second estimate using a binomial distribution, contains trial number (to account for learning or fatigue effects), starting side (to account for possible effects on left versus right initial presentation), SOA and first estimated duration, and a random effect for participants. The second model is identical apart from having SOA removed, representing an account in which SOA, and therefore multitasking-penalty, does not influence performance. A model comparison shows that the first model with SOA has indeed a significantly better fit to the data ( $\chi^2_{(1)}=5.61, p=0.018$ ), indicating that SOA does have an

effect on proportion of correct responses. The direction of the estimated effect ( $\hat{\beta} = -.0003$ ,  $z = -2.34$ ,  $p = 0.019$ ) implies that longer SOAs are associated with a lower accuracy on the second estimate, which is consistent with the single timer account, but in the *opposite direction* as predicted by a dual-tasking penalty account.

To test whether the first estimate has an influence on the second estimate, we again compared two LME models. The first model contains the second estimated duration as a function of the fixed effects of trial number, starting side, SOA and first estimated duration, and a random effect for subjects. The second model has the first estimated duration removed, but is otherwise identical. The fit of the second model is significantly worse ( $\chi^2_{(1)} = 316$ ,  $p < 0.001$ ), indicating a significant contribution of the first estimated duration for the prediction of the second estimated duration. The estimated effect ( $\hat{\beta} = .284$ ,  $t(3335) = 18.21$ ,  $p < 0.001$ ) indicates that longer first estimations yield longer second estimations. This is obviously in line with a single timer account, as the second estimation is dependent on the first. However, this could also be explained in the context of multiple timers as the amount of overlap influences both first and second estimations. This assumes a negative contribution of SOA on the second estimation: the shorter the SOA, the longer the overlap, and therefore the longer the estimated durations will be.

To assess the contribution of SOA on the second estimation, we constructed a similar model to the best fitting model described above, but in which SOA was removed. The fit of the reduced model is significantly worse ( $\chi^2_{(1)} = 347$ ,  $p < 0.001$ ), indicating a significant contribution of SOA in the prediction of the second estimate. However, the estimated effect of SOA ( $\hat{\beta} = .494$ ,  $t(3335) = 19.12$ ,  $p < 0.001$ ) indicates that longer SOAs yield longer second estimations (see Figure 5). Again, this estimated effect is in the *opposite* direction as predicted by the multiple-timers account, but is in line with the single timer with temporal discounting account.

## Summary

As the estimate of the first duration contributes significantly to the estimated second duration, and the estimated SOA effects are both significant and in the same direction as predicted by the single timer account, the conclusion from this experiment is that it is more probable that humans have only a single timer. However, this single timer can be used relatively efficiently to estimate multiple intervals, although performance of the second estimation is negatively influenced by the logarithmic scale of the time generating mechanism.

An effect unexplained by the single timer account is the lower level of performance on the first estimation. According to the single timer account, the proportion of correct *first* estimations in the two-intervals phase should be

similar to proportion of correct estimations in the training phase. As discussed above, the observed proportion was significantly lower during the two intervals phase than during training. This effect cannot be explained on the basis of timing alone. However, embedding a timing mechanism in a general cognitive architecture explains this effect elegantly, as we will discuss in the next section.

## Model

In Taatgen, Van Rijn and Anderson (in press), we have proposed a time mechanism that is embedded in the ACT-R general cognitive architecture (Anderson, 2007). We have shown that some of the phenomena traditionally described as pure timing phenomena, for example the effects of attention on cognitive timing, can better be explained as effects of the cognitive architecture or context on the task. The system contains a simple time generating system that consists of a single internal clock that generates logarithmically scaled pulses. These pulses can be read out by the cognitive architecture and stored in a declarative memory trace for later reuse. This approach explains, when embedded in an architecture that accounts for the retrieval of stored durations, both the basic findings associated with timing and more complex attention-demand related phenomena. Although ACT-R is a fairly complex theory, only a few components are crucial in understanding how the model can fit the experimental data presented here.

## Time estimation

The temporal module of ACT-R measure time in units that start at 100ms, but become gradually longer, creating a logarithmic representation of time, as illustrated in Figure 3 (see Taatgen, van Rijn & Anderson, in press, for details). This means that 2 seconds corresponds to 17 units or pulses in the temporal module, but 4 seconds only to 29 pulses instead of 34. Based on the initial single presentation of the intervals we assume participants have arrived at a reasonably stable internal representation of 2 seconds (17 pulses) at the start of the overlapping presentations. When the start signal for the first interval is given, the timer is started. After the SOA, the start signal for the second interval is given, prompting the model to store the value of the timer at that moment (in the examples 5 pulses for a 0.6 sec SOA and 13 pulses for a 1.5 sec SOA). When the timer reaches the 17 pulses, the value that corresponds to 2 seconds, the model will make the first response. It then adds the stored SOA value to 17, and waits until the timer reaches that value (i.e., either  $17 + 5 = 22$  or  $17 + 13 = 30$  pulses) to make the second response. As Figure 3 illustrates, the logarithmic scale introduces a bias in the second response that becomes larger with longer SOAs: The bias for the 0.6 sec SOA trial is  $2.73 - 2 - .6 = .13$  sec, for the 1.5 sec SOA trial, the bias is  $4.06 - 2 - 1.5 = .56$  sec.

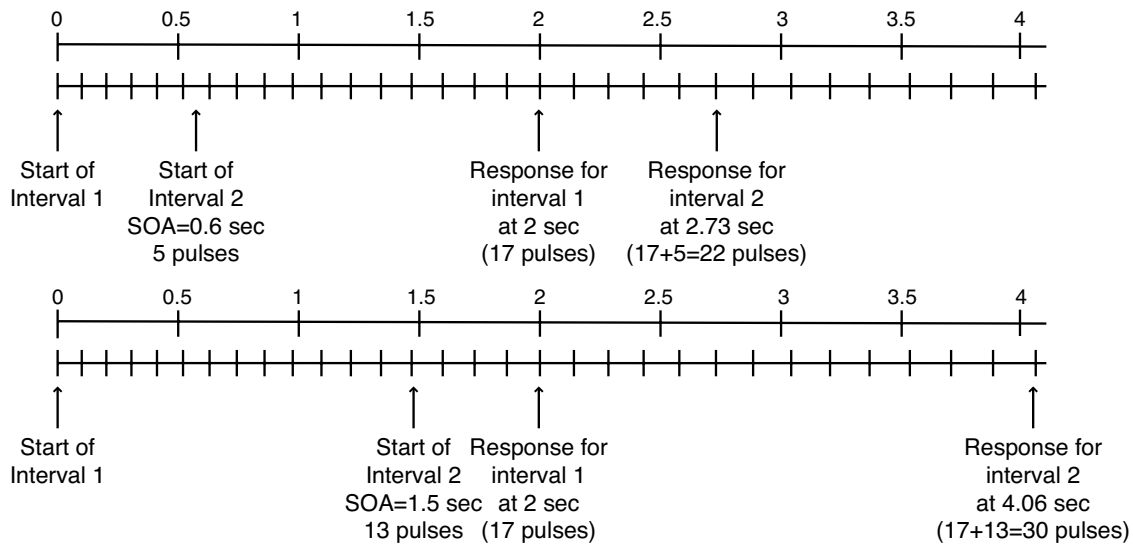


Figure 3. Illustration of the model for a short (0.6 sec) and a long (1.5 sec) SOA. The top line shows real time on a scale of seconds, while the lower line shows the subjective time in terms of logarithmically scaled pulses of the temporal module.

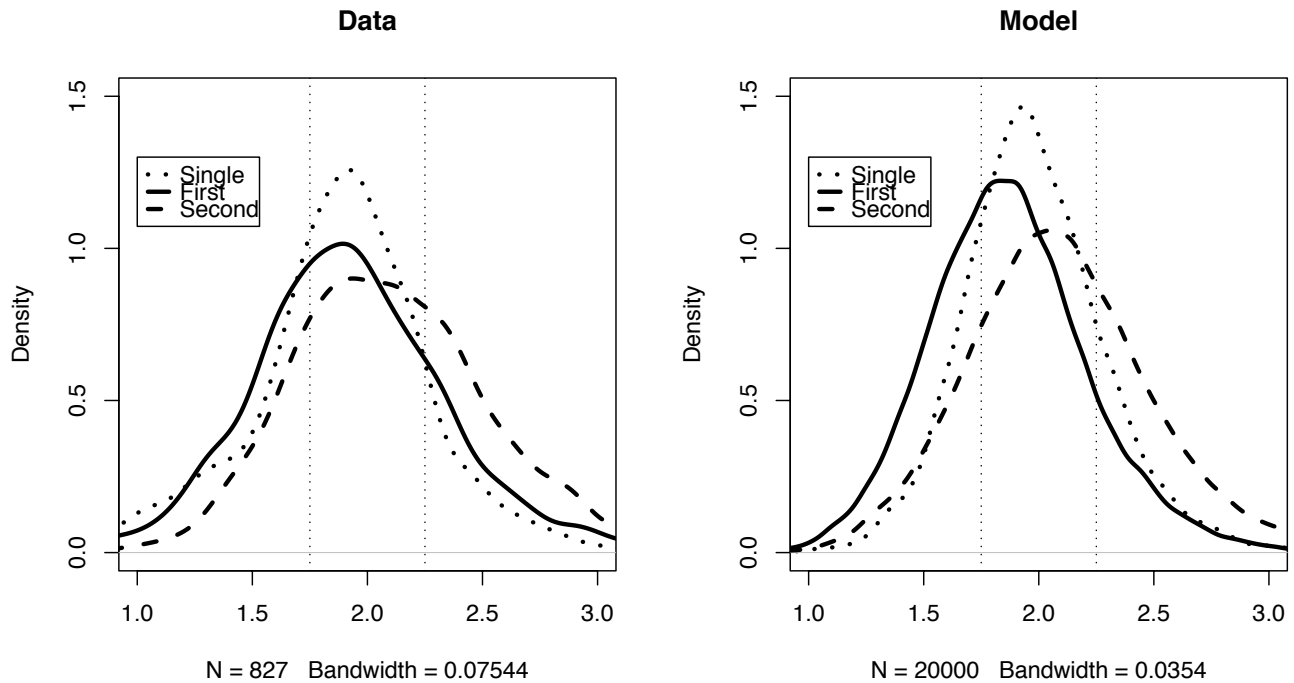


Figure 4. Distributions of estimations in the single task block, the first estimate in the dual task block and the second estimate in the dual task block. The vertical lines indicate the region of correct responses (1.75-2.25 sec).

### Representation of the time interval

The model maintains a representation of the time interval in its declarative memory. This representation is based on instance theory (Logan, 1988), which assumes that each

experience creates an example in memory. Each time the model produces an interval and receives positive feedback, it will create an instance in declarative memory for that interval. If the model receives “too late” as feedback, it will not store the instance, and will bias its next response by subtracting one or two pulses (randomly with equal

probability) from the next instance it retrieves. The reverse is true if the model receives “too early” as feedback.

During the single task block, the instances will average around 17 pulses, which corresponds to 2 seconds. However, during the dual-task block its response on the second estimate will often be too late because of the addition of the perceived SOA, leading to feedback that prompts the model to shorten its representation of the interval. This shorting will continue until accuracy on both intervals is approximately equal and the too earlier responses for the first estimation cancel out modification based on the too late second responses. However, performance will never be completely stable, because the SOA introduces extra variability in the second estimate that the model cannot fully compensate for.

### Model results

Figure 4 shows the distributions of time estimates for the first block in which only a single estimate had to be made, and the first and second response in the second block. It shows that second responses are generally later than first responses, which is due to the bias of the logarithmic scale. It also makes evident that the distribution of the first response is pushed somewhat to the left, corresponding to around 16 pulses, compared to 17 pulses in the single task case to compensate for the second response. This explains the decrease in accuracy on the first interval. Figure 5 shows the accuracies for both the model and the data. Consistent with Figure 4, the model performs slightly better than the participants, but shows the same overall effects, including an almost identical accuracy on the first and the second estimate in the dual-task condition.

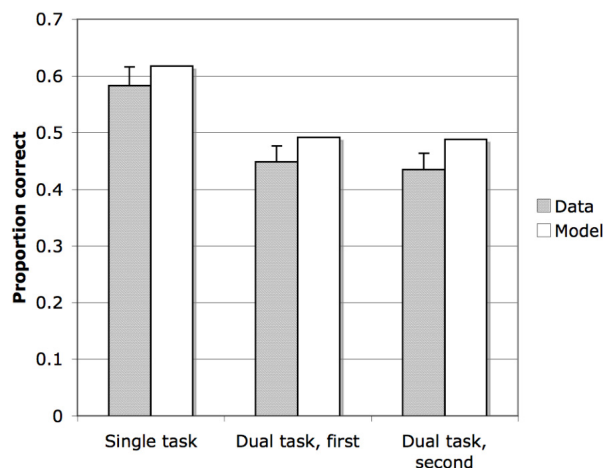


Figure 5. Comparison of accuracies for data and model

Figure 6 shows the effect of the SOA on the second response, and illustrates the effect of the logarithmic scale as the longer SOAs result in an overestimation of the second interval's duration.

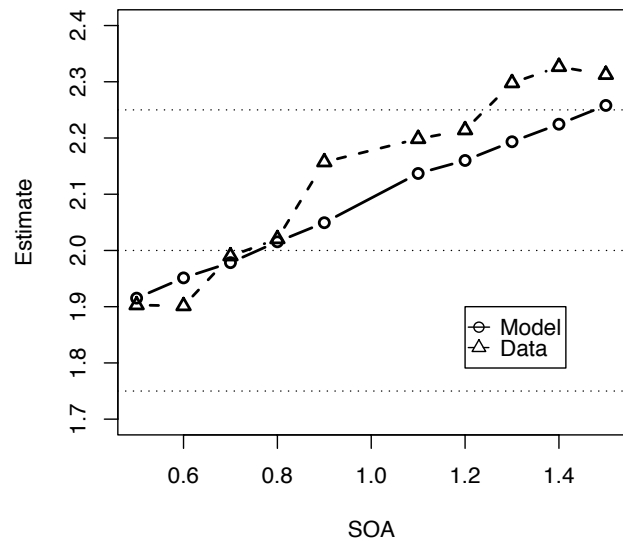


Figure 6. Effect of SOA on second estimate (model and empirical data)

### Discussion & Conclusion

Do multiple independent time generators drive human parallel time estimation, or do we strategically use the output of a single time mechanism for parallel time estimations? In this paper we presented an experiment that provides evidence for the latter account. Although all discussed analyses favor a single timer account, the most striking result is the positive relation between length of SOA and the estimated duration of the second interval. As the multiple-timers account predicts a negative effect, this positive relation can only be explained by strategically using the output of a single internal time generator.

However, the lower accuracy of the first estimate in the dual-timing phase compared to the single-timing phase cannot be explained by a theoretical analysis of the single-timer account. According to the single-timer account, performance of the first estimate in dual timing conditions should be equal to performance in single timing conditions, as processing the second estimation takes place after the first estimation is given. However, the presented computational model gives an elegant explanation of this effect. Because the two time intervals are estimated on the basis of a single main-estimation (resulting in the response for the first estimate), the feedback given for both estimations cannot be attributed to two different estimations. Thus, the model uses the feedback in a more general way, resulting in a shift towards earlier responses for both second and first estimations when the second estimation was too late (and vice versa). Because the logarithmic scale biases responses towards late responses, both estimations will shorten, yielding a lower accuracy for the first estimate in dual-time estimations than during single-time estimation.

An interesting aspect of the model is that it assumes that some form of temporal arithmetic is possible, at least for

very simple but non-trivial additions. We are currently setting up a new experiment to test the implications of this assumption. Important to note is that, although the model internally represents the time in terms of a number of pulses, the value of this number is assumed to be meaningless with respect to inspectability: being able to time a certain interval correctly does not imply that one can state how many pulses are associated with that interval.

Concluding, we have, as in Taatgen, Van Rijn, and Anderson (in press), showed in this paper that adding temporal processing capacities to ACT-R facilitates more precise explanations of what is necessary to keep the time.

### Acknowledgments

This work was supported by Office of Naval Research grant N00014-06-1-005.

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